

WCER Working Paper No. 2003-10
August 2003

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The research reported in this paper was supported by the Spencer Foundation, the National Center for Improving Student Learning and Achievement in Mathematics and Science under Grant No. R305A960007, and the Wisconsin Center for Education Research, School of Education, University of Wisconsin–Madison. Any opinions, findings, or conclusions expressed in this paper are those of the author and do not necessarily reflect the views of the funding agencies, WCER, or cooperating institutions.

Creating a Research Community in Mathematics Education¹

Thomas A. Romberg

In professional schools such as medicine, law, and education, the work of the faculty is directed toward the preparation of new professionals and the updating and refreshment of existing professionals rather than toward research. Similar expectations have long characterized the work of mathematics teacher educators. Typically, their focus lies in the teaching of content or methods courses for prospective teachers, the supervision of student teachers, the teaching of professional development courses, and so on. Although conducting research studies is often considered part of occupational work, the preparing of novices for the classroom leaves little time to address pedagogical or cognitive learning issues in any depth. As a result, research has rarely been the central focus of mathematics educators.² Mathematics education, therefore, is a young academic field, and research on the teaching and learning of mathematics is even younger. A research community in mathematics education has only gradually begun to emerge in the past half-century, and the process is still ongoing.

In 1969, when I chaired the preparation of an issue of the *Review of Educational Research* on mathematics education, we found over 1,000 research articles on the teaching and learning of mathematics that had been produced during the 1960s, but nearly all were the only papers prepared by an author during that decade, and the few authors of multiple papers, in almost all cases, were educational psychologists, not mathematics educators (Romberg, 1969). Although investigations were being carried out in the 1960s, there were almost no mathematics education scholars carrying out coherent ongoing research programs. As Jeremy Kilpatrick commented in his 1969 review of the research on problem solving for the *Review of Educational Research*, “problem solving is not now being investigated systematically by mathematics educators. Few studies build on previous research; few studies have an explicit theoretic rationale” (p. 523).

Increasingly since the 1960s, however, the primary job of a number of mathematics educators has involved research on the teaching and learning of mathematics in schools. As a consequence of the coherent research programs these scholars have produced, a mathematics education research community is emerging. Before examining the evidence that supports these claims, however, I want to emphasize the importance of the existence of research scholars *and* a research community in which they operate to the flourishing of research in such areas as the teaching and learning of mathematics in schools: In order for research to be productive and useful in any discipline, it must be conducted within a research community (Crane, 1972; Kuhn, 1970).

¹ A preliminary version of this paper was originally prepared for the Spencer Foundation’s 30th Anniversary Conference, “Traditions of Scholarship in Education,” held January 24–25, 2001, in Chicago. It has since appeared as a chapter in a book of the same title edited by E. C. Lagemann and published in 2002 by the Spencer Foundation. Reprinted by permission of the publisher.

² For rhetorical purposes, in this paper I arbitrarily separate mathematics educators into those who focus on teacher education and those who focus on research.

Researchers

The growth in the number of research faculty is important because the work of researchers differs from that of faculty involved in other aspects of education. Most people are curious, but few pursue the objects of their curiosity systematically and thoroughly. Researchers who devote their lives to understanding the teaching and learning of mathematics in school settings are insatiably curious about why humans behave in the ways they do in school and the conditions under which instruction is carried out. Most learn at an early age to accept the prevailing societal views of the ways schools and curricula are organized, the ways teachers and students behave, or ought to behave, in classrooms. It can come as a shock to find that the truths we accept as basic axioms of all schooling behaviors simply do not hold true in other cultures.

To illustrate, let me refer to a personal example. In 1979, I was part of a Fulbright team of U.S. education researchers invited by the Soviet Academy of Pedagogical Sciences to examine teaching and learning in their classrooms. In the first classroom I visited, I was surprised to observe that after a student had solved an assigned problem, she let another student copy her work. I commented about the “cheating” to a Russian colleague and was told, instead, that the student was “helping” (Romberg, 1979, p. 90). Curious about this “helping,” I verified the phenomenon in several classes. In the USSR at that time, it was believed that

schools should not try to differentiate between students. Being a group member of the collective; helping each other; not standing out or being different . . . are valued. . . . Collaboration is viewed as a socialist strategy to train students to be better members of collectives. (p. 91)

In other words, “helping” was a deliberate consequence of the way schooling was perceived in that society.

In looking at this example, three things should be noted. First, a scholar does not necessarily accept a puzzling phenomenon as an anomaly, but is curious and seeks to understand. Second, to reach an understanding of any phenomenon, a scholar must gather evidence—searching documents, asking others, or observing for oneself in a systematic manner. In each case, the methods of systematically searching for evidence and reporting on what is found is typically called “disciplined inquiry” (Cronbach & Suppes, 1969). To reach my own conclusions about “helping,” I read and reread several documents, asked colleagues and Soviet researchers lots of questions, and observed several more classes in several schools. I then marshaled the evidence and wrote a report of what I had found. No research is complete unless what is found is shared with others.

Scholarly investigation involves primarily normal reasoning, made rather more precise and exact: There is nothing mysterious about its methods. A researcher and an “ordinary” teacher-educator do not think about problems in entirely different ways, but the methods used by the researcher are intended both to be much more finely adjusted and to operate more sensitively and rigorously. My investigation of “helping” is an example of the way a scholar thinks and works: Scholars are trained to be skeptical about experience. One observation was not enough. I wanted a lot more evidence before making any assessment of what I had seen, before deciding, in this instance, that what I had seen was “helping,” not “cheating.” Note also that such statements of assessment are never considered “true,” but rather viewed as “reliable

propositions” that can be used to make practical decisions (e.g., if one wants to foster group cooperation, then teaching students to “help” each other might well prove productive).

The primary role of researchers is to provide our society with reliable evidence to back up claims. Too many people are inclined to accept whatever statements are first urged on them strongly, clearly, and repeatedly, even when such statements are dogmatic or outrun what is known with certainty. Similarly, most people are slow to acknowledge that the true measure of their knowledge is their awareness of how little they (and we) know with certainty. But knowing little or nothing is seldom an obstacle to being confident: People with intense convictions sometimes adopt and pursue bold policies with vigor and persistence, but with little or no evidence to support their beliefs. As Larabee (1945) put it, “anyone who has surveyed the long history of man’s claims about knowing is struck by the discrepancy between the pretentiousness of most knowledge claims and the small amount of evidence actually available with which to back them up” (p. 82). Researchers, in contrast, put forth claims that go beyond a mere opinion, guess, or flight of fancy. As Cronbach and Suppes (1969) noted,

Disciplined inquiry has a quality that distinguishes it from other sources of opinion and belief. The disciplined inquiry is conducted and reported in such a way that the argument can be painstakingly examined. The report does not depend for its appeal on the eloquence of the writer or on any surface plausibility. (p. 15)

By gathering evidence and constructing reasonable arguments, researchers substantiate conjectures. This process is arduous, and endless. Findings, once achieved, do not stay “finished” and complete: The evidential base—what constitutes a reasonable argument and the given purposes—continually changes. The evidential grounds on which I based my findings about “helping” in Soviet schools, for example, might well have changed since the fall of the Soviet system of government. Regardless of such changes in context, however, research is valued as a way of demonstrating *reliable* knowledge.

For scholars to produce reliable knowledge, there are at least three other aspects about their work that need to be understood (Popkewitz, 1984):

- Research is done in response to social and political needs.
- A researcher carries out studies as a member of a scholarly community.
- Academic freedom is essential.

Researchers are rarely free to investigate any phenomenon that strikes their fancy: They are constrained both by social pressure (which dictates which problems are worth studying and how resources are allocated) and by the expectations of other scholars about how problems are viewed, investigations carried out, and reports written. Although working under these constraints, researchers need to be free to investigate without the added demand to start from politically correct preconceptions, to use only politically correct approaches to problems, or to produce politically correct results.

In contrast to the early 1960s, today there are now a number of experienced scholars conducting systematic research, and that number is growing. Again in contrast to the 1960s, recent reviews of research on the teaching and learning of mathematics in schools include numerous examples of scholars who have published multiple papers with many colleagues and who represent coherent research programs (e.g., Grouws, 1992; National Council of Teachers of Mathematics [NCTM], 1994; Schoenfeld, 2001). These scholars include those trained as mathematicians who have become curious about education (e.g., Robert Davis, Jim Kaput, Alan Schoenfeld) and psychologists interested in schooling (e.g., Jim Greeno, Rich Lehrer, Lauren Resnick), but most of these scholars trained as mathematics educators (e.g., Debbie Ball, Tom Carpenter, Paul Cobb, Maggie Lampert, Less Steffe).³

Research Communities

As noted earlier, there is little evidence that a mathematics education research community existed in the 1960s, although the basis for such a community began to emerge at that time. Beginning in that decade, there was a gradual growth toward a concept of “normal science” (Kuhn, 1970) in mathematics education, evident in the research productivity on the teaching and learning of mathematics in schools growing out of the 1957 launch of Sputnik by the USSR. Swept by the enthusiasm for improving school mathematics, federal resources were made available for curriculum development, teacher retraining, and later, evaluation. Eventually, some resources in this “modern math” era were allocated to research. Although policymakers first complained about traditional practices to justify making changes, and later complained about the lack of impact of those reform efforts, it was only when someone asked for the data behind such complaints that the need for research was acknowledged and funded. In fact, not until the 1960s did the salesmanship related to “modern math” give way to questioning and, in some cases, careful inquiry.

In the 1960s, the complaints about school mathematics following the launch of Sputnik drove the developments and, later, the research in the field. The current reform effort is responding to more complaints about school mathematics. Arguments for change were posed in *A Nation at Risk* (National Commission on Excellence in Education, 1983) and *Educating Americans for the 21st Century* (National Science Board Commission on Precollege Education in Mathematics, Science, and Technology, 1983)—reports that focused on the evidence that schools were failing to educate students to be productive employees in the workplace. These reports claimed that schools, as constituted, were the products of an industrial era that had ended and that the school mathematics curriculum still reflected the industrial needs of the 1920s, not the workplace needs of this era. They argued that the content and structure of the mathematics curriculum should not operate to indoctrinate students with past values, but should be derived from visions of the future. The same basic argument that precollege mathematics is a local, state, and national concern has again been made in *Before It's Too Late: A Report to the Nation* (National Commission on Mathematics and Science Teaching for the 21st Century, 2000) and adopted by President Bush in the No Child Left Behind Act (2002). The concern continues to be that the nation depends on a strong, competitive science and engineering workforce and a

³ I have made no attempt here to be inclusive: The individuals I list are but a few of the scholars who have published papers in the past decade.

citizenry equipped to function in today's complex world. Responsibility for providing evidence to support the claims in these reports, and later to provide data on the impact of the changed policies and practices, lies in large part with this newly emerging research community in mathematics education.

The Emerging Research Community

Each field of scholarship has particular constellations of questions, methods, and procedures, which provide shared ways of seeing the world, of working, of testing each other's conjectures, and so forth. Research communities, as they emerge, make commitments to certain lines and ways of reasoning and premises for certifying knowledge. Kuhn (1970) referred to the work and products of research communities as "normal science" in that a group of scholars have agreed on a set of phenomena, have chosen a language to describe it, have accepted an approach to test conjectures about it, have agreed to its utility, and have shared their ideas, data, findings, and reports. The investigations the group conducts should be seen as embodying some of the elements of a craft—directing attention both to the personal autonomy and the collective responsibility of the group in creating and testing conjectures. Being a member of that community, therefore, consists of more than learning the content of the field: It is also about learning how to see, think about, and act toward the world from the structural context of that field. The assumption, supported by Crane's studies (1972) of research growth, is that progress in understanding a field of inquiry occurs when research communities are formed.

The apparent lack of such a mathematics education community in the 1960s and its recent emergence can perhaps best be seen by contrasting the six more or less defining features of a research community in any field:

1. A shared purpose and vision
2. Availability of resources
3. A means of sharing information
4. Agreed-on theoretical notions
5. Acceptance of research in the field
6. A means of training new participants

A Shared Purpose and Vision

The purpose of research in any discipline is, first, to understand some phenomenon; second, to interpret that understanding in terms of theoretical notions, and finally, to improve practice or make predictions based on the knowledge gained from that research. To capture the interrelationship of the components in the schooling process, I have used Begle's (1961) diagram shown in Figure 1 (Romberg, 1992a): The enterprise of schooling is situated within a social context; the mathematics curriculum involves a subset of mathematics; and a teacher carries out

instruction with a group of students within a school classroom over time. Each of these components needs to be considered as one examines an existing system or designs a new system.

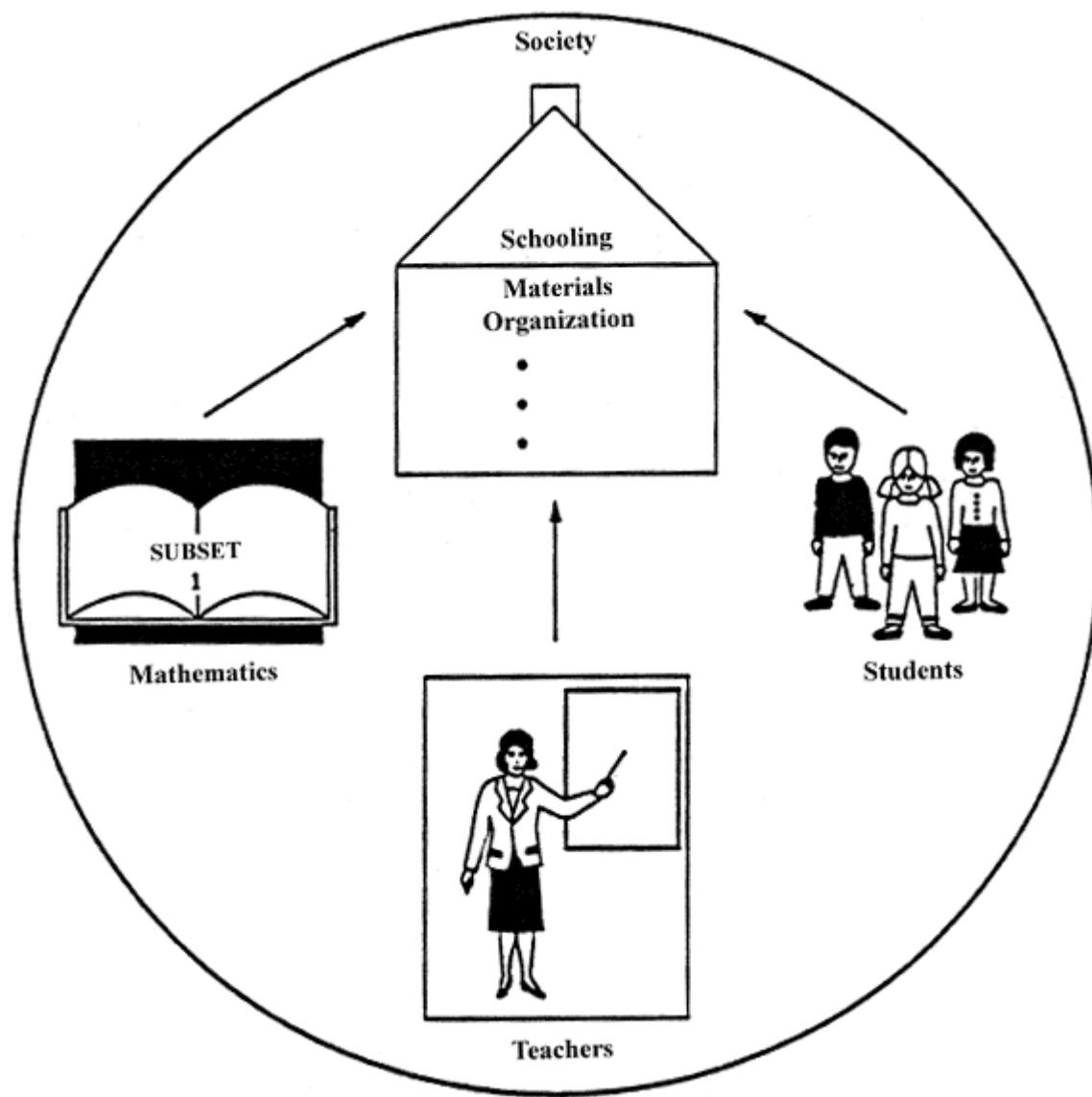


Figure 1. Components in the schooling process (Begle, 1961).

In the post-Sputnik era, the primary reform efforts involved mathematicians' attempts to change the mathematical content in the texts and provide courses to improve the mathematical background of mathematics teachers, on the one hand, and psychologists' attempts to understand mathematical abilities, on the other. At that time there was a belief that the curriculum and the way it was taught needed to change. The vision that drove these efforts was based on the assumption that using structural features of mathematics as the basis of the curriculum, providing teachers information about such mathematical features, and developing knowledge about abilities would better prepare the mathematically able for college and, potentially, careers in mathematics, science, engineering, and so on (for more details, see Begle, 1970). It should be

noted that this was not a systemic attempt to change school mathematics for all students, nor was there evidence that any of these efforts would accomplish this goal. Furthermore, these reformers were unaware of the coherence of the traditional system of mathematics instruction and what it would take to change it (Schrag, 1981). Only when policymakers began to question the effectiveness of these efforts was research called for and systemic programs of inquiry begun as an attempt to make sense of the complexity and stability of schooling in that era.

Similarly, today the current research agenda in mathematics education is focused on the current reform vision for school mathematics. (For background information on the reform movement, see Mathematical Sciences Education Board, 1989, 1990; NCTM, 1989, 1991, 1995, 2000; Romberg, 1992b). This standards-based reform vision is based on the perceived consequences of several related factors: the development of new technologies; changes in mathematics itself; changes in the use and application of mathematics; new knowledge about learning, teaching, and schools as sociopolitical institutions; and renewed calls for equity in learning mathematics, regardless of race, class, gender, or ethnicity. This time, we understand that the current school mathematics operates within a coherent system—reform will occur only if an equally coherent system replaces it. Research can provide information needed to understand and to help design such a system.

In contrast to the post-Sputnik reform efforts, the current movement is based on information generated from research on the teaching and learning of mathematics in relationship to the education components in Figure 1. First, mathematics is not a static discipline composed of a large collection of concepts and skills to be mastered in some designated order. Instead, it must be seen as a dynamic discipline composed of interrelated sets of signs and symbols, rules for the use of those representations, problem situations that have given rise to the invention of those signs and symbols, and strategies for investigating and solving problems that can be represented by those signs and symbols. Furthermore, the availability and uses of new technologies are changing, in fundamental ways, the problem situations, methods of representation, and strategies used.

Second, the learning process for every student involves the cognitive construction of ideas via social interactions that occur both in and out of school. This cognitive perspective has been used to describe and clarify the processes of learning when students formulate, represent, reason about, and solve problems in specific content domains. This perspective about learning centers on the notion that learning mathematics involves “doing” mathematics. One does mathematics by abstracting, inventing, proving, and applying. Students then construct mathematical knowledge from these purposeful activities. The eminent mathematician George Polya (attributed by Kilpatrick, 1987) felt that we understand mathematics best when we see it being born, by either following in the steps of historical discoveries or by engaging in discoveries ourselves.

Third, instruction should focus on making classrooms discourse communities. Students need to share their ideas, reasons, and strategies with other students in an open, supportive environment. And teachers must not just *listen* but also *hear* what students say. Teachers and students alike must recognize that not all mathematics is learned in a classroom setting. Students must be encouraged to make links between real-world mathematics and classroom mathematics:

They should be encouraged to seek out applications of mathematics in the world around them and to continue their sense-making and mathematical construction outside the classroom.

Fourth, teachers who provide an environment that promotes deep thinking need a deep understanding of the mathematics appropriate to the instructional level at which they teach. This understanding must include knowledge in the mathematics being taught and in the ways it is related to the mathematics prior to and beyond the instructional level of students; knowledge of the ways students reason about and come to understand the mathematics at their instructional level; and knowledge of the nature of mathematical activity. It is this knowledge that allows teachers to competently guide their students through the twists and turns of rich mathematical discourse.

In contrast to the efforts of the “modern math” era, today the long-term purpose of research on the teaching and learning of mathematics is rooted in the desire to replace the current coherent system of school mathematics with an equally coherent system. The vision of how this could be accomplished is based on changed notions of what it means to understand mathematics, how it is learned, and how it should be taught.

Availability of Resources

Undoubtedly the creation and maintenance of a research community is dependent on the availability of research grants, which in turn have been made available from federal agencies and private foundations. Although the U.S. Department of Education was first empowered to fund research projects via passage of the Cooperative Research Act in 1954, it was not until 1963, with the passage of the Elementary and Secondary Education Act (ESEA), that significant funds were made available for education research. In particular, research centers were established to conduct programmatic research, and evaluations of the products of development projects were required. Although none of the new centers were dedicated to the teaching and learning of mathematics, some work on school mathematics was carried out in several of the centers (notably the centers at Wisconsin, Pittsburgh, and Stanford). At the same time, the National Science Foundation began to request formal evaluations of student performance for the various development programs it was funding.

A notable exception was the National Longitudinal Study of Mathematical Abilities (NLSMA). In 1962, when calls for data on the impact of the post-Sputnik “modern math” materials were made, the National Science Foundation funded the School Mathematics Study Group (directed by Professor Begle at Stanford University) to conduct NLSMA. This was the first major study of school mathematics funded by a federal agency. The purpose of this “large-scale, five-year research program was to gather necessary achievement data for the constantly evolving processes of mathematics curriculum revision and modification” (Cahen, 1965). Over 110,000 students participated in this study, which attempted not only to assess achievement, but also to learn more about the nature of mathematical abilities. Although mathematicians and psychologists directed the study and the results were of little interest to policymakers by the time the data were collected, it became the training ground for a number of doctoral students in mathematics education (notably Jeremy Kilpatrick, Jim Wilson, Jerry Becker, Ray Carry, and

myself) who later became researchers at major institutions and influential contributors to the current reform efforts.

Since that time, federal funding for research in mathematics education has varied, with support gradually increasing until 1980, when funding was dramatically cut. In the 1970s, several private foundations started education research grants, some of which targeted school mathematics. Notably, in 1971, the Spencer Foundation started such a grant program, which has since supported over 30 major studies on mathematics education. Other foundations that periodically have supported such research include the Ford Foundation and the Carnegie Foundation. Following the publication of *A Nation at Risk* in 1983, funding of research on the teaching and learning of mathematics in schools again increased. In particular, in 1986 the U.S. Department of Education first funded a research center dedicated to mathematics education at the University of Wisconsin–Madison. Since then, that center has been a major contributor to the current reform efforts. Although the amount of resources to conduct research on the teaching and learning of mathematics has fluctuated and remains meager, without such resources, little research on the teaching and learning of mathematics in schools would have been conducted.

Means of Sharing Information

Crane (1972) argued that both formal and informal means of communication are critical components of research communities. Only by regularly sharing information do scholars know what is happening in a field and if their research will have any impact, but to share ideas about the teaching and learning of mathematics in classrooms, scholars need to communicate with each other on a regular basis. Barton and Wilder (1964) found that the lack of interpersonal communication seriously weakened the intellectual development in applied reading research. Productive reciprocal sharing usually occurs via a variety of informal means: letters sent to friends asking for comments about ideas, requests to colleagues to read and react to drafts of papers or reports, and meetings and conversations at professional conferences.

By the mid-1960s, a few scholars had begun conducting research on the teaching and learning of mathematics. However, research in the field lacked coherence, and investigators lacked professional identity. At that time, no organized meetings on research were regularly held, and only two publications of NCTM, the *Arithmetic Teacher* and the *Mathematics Teacher*, along with *School Science and Mathematics*, served as the main outlets for reporting research studies. Such reports, however, were viewed by many on the NCTM Board of Directors as inappropriate for teachers. In 1964, based on the recommendations of an ad-hoc committee, NCTM organized a Research Advisory Committee, which in 1968 recommended that a research presession be held in conjunction with the organization's annual meeting and that research sessions be established within the annual meeting. With some reluctance, NCTM initiated this practice in 1969, and it has now become a standard feature of the organization's annual meetings. At the same time within the American Educational Research Association (AERA), a newly formed special interest group for mathematics education sponsored paper-reading sessions and a symposium at the 1969 AERA annual meeting. Today, there are over 300 members of this group.

In 1966, concerned mathematicians established the International Commission on Mathematical Instruction (ICMI), which in turn convened the first International Congress on Mathematics Education (ICME) in 1968, with meetings to be held every 4 years. Initially, concerns concentrated on the mathematical content of school mathematics. At the 1976 meeting, however, a working group composed of mathematicians and psychologists focused on education research. A member of the group, Hans Freudenthal, later that year hosted a meeting at which the International Group for the Psychology of Mathematics Education (PME) was organized. Annual meetings of PME have been held every year since, and several local groups (e.g., PME–North America) have been formed.

As Johnson, Scandura, and I (Johnson, Romberg, & Scandura, 1994) described in the *Journal for Research in Mathematics Education*'s 25th Anniversary Special Issue, in 1967 the same Research Advisory Committee that persuaded NCTM to start research reporting sessions at its annual meeting produced an initial issue of a research journal and recommended that NCTM publish a mathematics education research quarterly on a 3-year trial basis. As a consequence, NCTM's *Journal for Research in Mathematics Education* was inaugurated in 1969. At about the same time, Freudenthal started *Educational Studies in Mathematics* (first issue in May 1968) for mathematics educators. These two are now the premier journals that publish studies on the teaching and learning of mathematics. Several other journals have been started, many in other countries, reflecting the growing emphasis on mathematics education research around the world.

As a result of this activity, researchers studying the teaching and learning of mathematics in schools today, in contrast to researchers in the 1960s, have ample opportunities to regularly share their ideas with other researchers in both informal and formal ways.

Agreed-On Theoretical Notions

Merton (1957) argued that there is a strong relationship between the particular structure and goals of research groups, the general worldview that they accept, and the productivity of the group. Initially, group members communicate in the language of everyday experience. Highly developed groups tend to express their ideas in technical terminology, in part because scholars require a language suitable for precise communication. One way, therefore, of determining if a mathematics education research community exists is to see if there is a shared language—to see whether scholars agree on theoretical notions underlying the teaching of mathematics. Let's return to the four components in Figure 1 (mathematics curriculum, student learning, teachers and instruction, schooling with a social setting), add “research methods,” and discuss each in terms of the contrast between the perspectives of the 1960s and those of today.

Mathematics curriculum. To the Romans, a curriculum was a rutted course that guided the path of two-wheeled chariots. The Mathematical Sciences Education Board (1990) returned to this original meaning of the word when it described curriculum as a path to be followed—an appropriate metaphor for school mathematics in the 1960s and even in many schools today. This path involved arithmetic through Grade 7 or 8, followed by a year of algebra, a year of geometry, and a year of advanced algebra and trigonometry. Unfortunately, most students failed to complete that path. As Schoenfeld (2001) pointed out, historically from Grade 9 on, there “was roughly a 50 percent attrition rate in mathematics each year” (p. 247). This conventional

curriculum followed a deeply rutted path, directed more by events of the past than the changing needs of the present. A vast number of learning objectives, each associated with pedagogical strategies, served as mileposts along the trail mapped by texts from kindergarten to 12th grade. Problems were solved not by observing and responding to the natural landscape through which the mathematics curriculum passed, but by mastering time-tested routines conveniently placed along the path. Scheffler (1975) denounced this perspective as follows:

It is no wonder that this [mechanical] conception isolates mathematics from other subjects, since what is here described is not so much a form of thinking as a substitute for thinking. The process of calculation or computation only involves the deployment of a set routine with no room for ingenuity or flair, no place for guesswork or surprise, no chance for discovery, no need for the human being, in fact. (p. 184)

What mathematicians were challenging in the “modern math” era was the structure of this path. Freudenthal (1968) described their efforts:

Mathematics is distinguished from other teaching subjects by the fact that, even in its actual totality, it is a comparatively small body of knowledge, of such generality that it applies to a richer variety of situations than any other teaching subject. Modern mathematics can be seen as an effort to reduce this body of knowledge even more and to enhance the flexibility of what remains to be taught. (p. 5)

Undoubtedly, the most abstract mathematics is the most flexible, but it is wasted on those who are not able to understand and use this flexibility. Students in “modern math” courses were taught abstract notations and structural properties (e.g., sets, groups, rings, fields) without context or relation to their use, but in the hope that they would be able to apply those properties and notations as needed. But neither the properties nor the notations have value unless they are used unequivocally to describe a commonly held construct. Although mathematics can be unambiguous, the inherent abstraction of mathematics was compounded by this “modern math” emphasis on notations rather than on understanding. To a novice or a learner, both the properties and the notations lay themselves open to multiple misconceptions. Eventually, teachers and the public at large rejected the “modern math” approach to school mathematics, and in the 1970s, in response to calls to go “back to the basics,” the curriculum returned to the past, less abstract mechanistic metaphor.

In contrast, today within the mathematical sciences community there is considerable agreement about what aspects of mathematics students should have an opportunity to learn. Three underlying notions about mathematics are considered central to the standards-based movement:

1. Mathematics as a socially constructed discipline
2. Mathematical literacy
3. Domain knowledge strategy

Philosophically, for centuries “mathematics has been viewed as a body of infallible truth far removed from the affairs and values of humanity” (Romberg, 1992b, p. 751). If it is

acknowledged that mathematics is “fallible, changing, and like any other body of knowledge, the product of human inventiveness” (Ernest, 1991, p. xi), then it is a continually expanding field of human creation and invention, not a finished product. Approaching mathematics as a social construct suggests that instruction needs to include the empowerment of learners to create their own mathematical knowledge. One way to reconsider mathematics in this manner is to agree with Ernest that “the basis of mathematical knowledge is linguistic knowledge, conventions, and rules, and language is a social construction” (p. 42).

In 1986, the Board of Directors of the National Council of Teachers of Mathematics (NCTM) established the Commission on Standards for School Mathematics to

create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields; and create a set of standards to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision. (NCTM, 1989, p. 1)

That students should become mathematically literate is the basic premise of the reform movement. In James Gee’s *Preamble to a Literacy Program* (1998), the term *literacy* referred to the human use of language. The ability to read, write, listen, and speak a language is the most important tool through which human social activity is mediated. Each human language and each human use of language has an intricate *design* tied in complex ways to a variety of *functions*. For a person to be literate in a language implies that the person knows many of the design resources of the language and is able to use those resources for several different social functions. Analogously, considering mathematics as a language implies that to be literate students not only must learn the concepts and procedures of mathematics (its design features), they must also learn to use such ideas to solve nonroutine problems and learn to mathematize in a variety of situations (its social functions). Philosophically, emphasizing mathematical literacy involves an epistemological shift from considering mathematics as a large set of concepts and skills to be mastered in a prescribed order and from judging student learning in terms of mastery of concepts and procedures, to considering mathematics as a collection of tools invented to make sense of the world and making judgments about students’ understanding of the concepts and procedures as well as their ability to mathematize problem situations. In the past, too little instructional emphasis was placed on understanding, and the tests used to judge learning failed to adequately provide evidence about students’ understanding or ability to solve nonroutine problems.

In contrast to the mechanical metaphor underlying traditional mathematics, the contemporary standards-based approach emphasizes the organization of concepts and procedures in mathematics, referred to as *domain-specific knowledge*. Although a few mathematical topics not now in the curriculum will be emphasized (e.g., statistics, mathematical modeling), most of the topics in the curriculum will remain, but organized differently. Knowledge in any field must be organized into several related domains or “conceptual fields” (Vergnaud, 1983). This domain view differs from the mechanical view in at least two important ways. First, the emphasis is not on the parts of which things are made but rather on the whole of which they are part. And second, this conception does not rest on a deterministic base of fundamental parts from which everything else is created. Instead, it rests on the signs, symbols, terms, and rules for use—the design features of a language that humans invent to communicate with each other. From this

perspective, Greeno (1991) proposed a different metaphor. Students develop understanding when a domain is

thought of as an environment, with resources at various places in the domain. In this metaphor, knowing is knowing your way around in the environment and knowing how to use its resources. This includes knowing what resources are available in the environment as well as being able to find and use those resources for understanding and reasoning. Knowing includes interactions with the environment in its own terms—exploring the territory, appreciating its scenery, and understanding how its various components interact. Knowing the domain also includes knowing what resources are in the environment that can be used to support your individual and social activities and the ability to recognize, find, and use those resources productively. Learning the domain, in this view, is analogous to learning to live in an environment: learning your way around, learning what resources are available, and learning how to use those resources in conducting your activities productively and enjoyably. (p. 175)

Elsewhere I have addressed the problem of engineering a curriculum based on this domain knowledge view (see Romberg, 1992b; see also de Lange, 1987). To describe a domain, one first needs to determine its formal aspects by identifying the symbolic statements that characterize it, specifying the implied tasks to be carried out, and specifying the rules that can be followed to represent, transform, and carry out procedures to complete the tasks. Doing this yields a map (a tightly connected network) of the domain. Such maps specify the key features and resources of the domain that are important for students to find, use, or even invent for themselves. Then what is needed is a collection of problem situations that engage students so that they explore each domain in a structured manner. Creating engaging problems is not easy. Although there is no doubt that many interesting activities exist or can be created, whether they lead anywhere is a serious question. Keitel (1987) argued that an activity approach in mathematics might lead to “no mathematics at all.” I have also written elsewhere (Romberg, 1992b):

Too often a problem is judged to be relevant through the eyes of adults, not children. Also, this perception is undoubtedly a Western, middle-class, static vision. Concrete situations, by themselves, do not guarantee that students will see relevance to their worlds, they may not be relevant for *all* students nor prepare them to deal with a changing, dynamic world. (p. 778)

Nevertheless, creating a set of situations that give meaning to the concepts and rules in a domain is as important as learning to follow procedural rules in that domain. In fact, it is assumed that becoming acquainted with a rich variety of situations is the means by which the concepts and procedural rules are understood. McDonald and Naso (1986) argued that students

construct new knowledge by noticing and analyzing experience, by filtering the experience through an interpretive network of previously learned concepts, and by readjusting this network in the light of new experiences. . . . This requires intensive engagement with phenomena, sufficient time for reflection, encouragement to risk new thinking, and support for an experienced teacher who can point out discrepancies, pose questions, and guide the learner’s thinking. (p. 2)

Although all the situations for which the domain could be used can never be considered (i.e., new situations are always arising), students should understand that the mathematical symbols

and rules are not idiosyncratic to a given situation. Only by doing so, will students understand that the symbols can be used to represent lots of seemingly dissimilar situations.

In summary, there is a need to change the curriculum metaphor from “following a path” to “exploring an environment.” In the “modern math” era, mathematicians tried to change the path by focusing on abstract structural properties. Today, the curriculum is undergoing reform based on notions that emphasize mathematics as a socially constructed discipline, mathematical literacy, and the domain knowledge strategy.

Student learning. In the 1960s, educational psychology was dominated by two traditions: (a) behaviorism as a means of shaping student performance and (b) aptitude testing as a means of identifying a student’s abilities. Behaviorism developed in the early years of the 20th century as psychologists studied what could be observed about the mental functioning of humans. Researchers emphasized external stimuli and responses and deemphasized the internal, nonobservable aspects of mental functioning. Thus, training for competent performance became the basis for learning. Behaviorism also became the dominant basis for mathematics instruction following the publication of E. L. Thorndike’s *The Psychology of Arithmetic* in 1922. This approach led to the segmentation of information into subjects, courses, topics, and so forth, eventually getting down to its smallest parts—behavioral objectives. The objectives were then related via hierarchies and mechanized via textbooks, worksheets, and tests. Without belaboring the argument, it should also be apparent that the process of instruction was viewed mechanistically. Lessons were related to specific behavioral objectives, sequenced via the hierarchies of objectives, and instructional tasks involved drill, practice, and reinforcements.

Although the development of ability tests also began at the turn of the 20th century in an attempt to identify mentally challenged children, the consequences led to the psychological testing movement. Several types of tests were developed, including tests to measure intelligence, scholastic aptitudes, and special aptitudes (e.g., mechanical, clerical, musical, artistic). Furthermore, the quantification of such traits led to the statistical investigation of intellectual trait organization (Anastasi, 1954). Such investigations were seen as lending scientific legitimacy to efforts to identify traits such as “mathematical abilities.” For example, the National Longitudinal Study of Mathematical Abilities (Cahen, 1965) administered a large set of psychological tests in an attempt to identify the traits of talented mathematics students. Another example is the series of studies by Julian Stanley and his colleagues on “mathematically precocious youth.” These studies utilized a battery of tests, including the Scholastic Aptitude Tests, to identify subjects; and subsequent follow-up studies culminated in a description of mathematically talented boys (Stanley, Keating, & Fox, 1974). It should be noted that interest in abilities has waned since that era. Even the A in SAT now stands for *achievement* rather than *aptitude*.

Since the 1960s, there has also been a revolution in the psychology of learning. This revolution had two roots. The first, *cognitive science*, had its birth in 1956, according to George Miller (1979), at the Symposium on Information Theory held at the Massachusetts Institute of Technology. This component of the revolution brought the mind and its method of operation into focus by using information-processing metaphors and language from the computer sciences (see Carver & Klahr, 2001, for the current status of this aspect of the psychological revolution; Gardner, 1985, for the history and development). The behaviorists had viewed the mind as a

“black box”; cognitive scientists attempted to unpack the box by studying long- and short-term memory capacity, expert knowledge storage and retrieval, and, recently, even neural processing.

A good example of a research program that reflects this perspective is Cognitively Guided Instruction (CGI), a project funded by the National Science Foundation and directed by Tom Carpenter and Elizabeth Fennema. An extensive body of research documents that children bring to instruction a great deal of informal knowledge that can serve as a basis for developing much of the elementary mathematics curriculum. The goal of CGI is to help teachers better understand children’s thinking so that they can build on that knowledge. Specifically, CGI was developed from a research-based analysis of the ways children usually think about number concepts and operations, focusing on the informal or invented strategies that children construct for solving problems. In CGI instruction, children initially directly model the action or relationships in problems, but over time, their strategies become more abstract and efficient. Place-value concepts and multidigit operations are conceived as natural extensions of the processes used to solve problems with smaller numbers and follow a similar pattern of development. By developing a deep understanding of the ways children’s thinking develops in a specific domain, teachers come to appreciate how children construct knowledge and to recognize that fundamental changes in their instructional practice are needed. Although in this program the analysis of children’s thinking focuses on specific content domains, it also provides teachers a framework for understanding mathematics and children’s thinking more broadly (Carpenter, Fennema, & Franke, 1996). Similarly, although the goal of CGI is to help teachers better understand children’s thinking so that they can build on children’s informal knowledge, the end purpose is not only to help teachers teach better, but also to help teachers make fundamental changes in their basic epistemology and practice.

The second root of the revolution in psychology, *social constructivism*, began as a consequence of the 1963 publication of *The Developmental Psychology of Jean Piaget* by John Flavell. Piaget’s perspective on cognitive development initially brought to psychologists a set of ideas and a language (e.g., *schema*, *assimilation*, and *accommodation*) to describe how the mind works and develops over time. Only later did scholars recognize that from this perspective, learning is a constructive process of selective reinvention and negotiation in social settings. (See Steffe & Kieren, 1994, for a discussion of the implications of constructivism on the learning of mathematics.) This constructive process, when coupled with ideas of Lev Vygotsky about the social formation of the way the mind works, led to today’s notions of social constructivism (Wertsch, 1985). From this perspective, there is “a sharp distinction drawn between teaching and training. The first aims at generating *understanding*, the second at *competent performance*” (von Glaserfeld, 1991, p. xvi).

A good example of a research program from this second perspective is the set of studies directed by Les Steffe that focused on how children construct personal mathematical concepts and procedures (Steffe & Cobb, 1988). The models developed grew out of interpreting children’s actions in particular environments. The children’s thoughts and actions are seen as “arising out of reflective abstraction on action and interaction in a world that allow the person to bring about or construct. In that sense the person’s knowledge structures and world of action out of which they rise are co-implicative” (Steffe & Kieren, 1994, p. 728).

Today, most mathematics educators no longer consider behaviorism a viable basis for the study of the teaching and learning of mathematics in schools. However, scholars are split between those who follow the ideas of cognitive science and those who consider themselves social constructivists. Nevertheless, there is agreement in both camps that critical learning of mathematics by students occurs as a consequence of building on prior knowledge via purposeful engagement in activities and discourse with other students and teachers in classrooms. Research carried out in the last decade has shown that in classrooms where the emphasis of instruction shifted from mastery of facts and skills to understanding, students became motivated to learn, and achievement at all levels increased.

Carpenter and Lehrer (1999) characterized understanding in terms of five interrelated forms of mental activity from which mathematical and scientific understanding emerges:

- Constructing relationships
- Extending and applying mathematical and scientific knowledge
- Reflecting on mathematical and scientific experiences
- Articulating what one knows
- Making mathematical and scientific knowledge one's own

Furthermore, because all learning occurs as a consequence of experiences, and all humans have a variety of experiences, virtually all complex ideas in mathematics are understood by individual students at a number of different levels in quite different ways. A student's level of understanding will change as a consequence of instructional experiences. For mathematics educators, therefore, the challenge is how to create classroom experiences that help a student's understanding grow over time. As recently stated in *How People Learn: Bridging Research Into Practice* (Donovan, Bransford, & Pellegrino, 1999):

Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classrooms (p. 10).

Teachers and instruction. In the 1960s, the daily pattern of instruction placed emphasis on the social management of a group of students, the importance of “coverage” of the school’s curriculum at a particular grade, and the role of the textbook. This pattern continues in many classes today. Weller (1991), in his study of traditional classrooms, found a common daily pattern of instruction in all classes: “It was evident that a repeating pattern of instruction occurred, which consisted of three distinctive segments: a review, presentation, and study/assistance period. This ‘rhythm of instruction’ was not unplanned or coincidental” (p. 128). This type of teaching developed so that a small number of adults (teachers and others) could organize and control a large number of students and “cover” the curriculum: Teachers and students needed to traverse a portion of the curricular path in a given time period. The Holmes Group (1986) described this naïve view of teaching as “‘passing on’ a substantive body of

knowledge . . . [and] ‘planning, presenting, and keeping order’ The teachers’ responsibility basically ends when they have told students what they must remember to know and do” (pp. 27–28). This succinct, if simplistic, characterization described a perception of the role and work of teachers at the height of the Industrial Age.

Weller (1991) characterized mathematics classes as follows:

A goal of mathematics teachers was to cover a prescribed amount of material preparing students to enter the next level of mathematics study. The overall pace of instruction required the [teachers] to teach the textbook, cover-to-cover, as the mathematics curriculum required. The sequential order of concept presentation was determined by the textbook editor and thus embraced by the department. (p. 128)

Later on, Weller (1991) argued: “Because coverage was a primary concern, lessons focused upon the transmission of bits of factual knowledge in the most expedient fashion” (p. 204). He then went on to point out that the path students were to follow was dictated by the choice of textbook used in the class. In fact, he argued that “the expert knowledge of the teacher was deliberately subjugated to that of the textbook. As a result of that process, the teacher was able to camouflage her role as authoritarian, thus eliminating student challenges of authority” (p. 133). These descriptions of the daily pattern, management, coverage, and role of textbooks are familiar to teachers, school administrators, and the public at large, and are well documented in the research literature. They represent features of a mechanistic form of organizational management considered appropriate for routine work (Perrow, 1967).

Weller’s (1991) description also reflects the findings of studies about “teaching as a nonroutine task.” Jackson (1986), Rowan, Raudenbush, and Cheong (1993), Stodolsky (1988), and others found that teachers with different disciplinary specializations show differences in their perceptions of teaching as a nonroutine task. In particular, teachers of mathematics and science see their subjects as more codified and routine in form than do teachers of English or social studies.

Today, the focus of schooling in reform classrooms has shifted from telling to listening and guiding in a discourse community, with assessment of student performance similarly shifting from counting the number of correct, quickly derived answers to small tasks, to making judgments based on student reasoning, strategies used, justifications, and so on. An initial way to characterize this change in teaching practices is as a shift from a “mechanistic” perspective to an “explore the environment” perspective, which sees teaching as involving nonroutine tasks, as in this example describing the reaction of a group of experienced teachers who were teaching a new unit on statistics:

The surprise came when we tried to teach the first lesson. There was little to “teach”; rather, the students had to read the map, read the keys, read the questions, determine what they were being asked to do, decide which piece of information from the map could be used to help them do this, and finally, decide what mathematics skills they needed, if any, in answering the question. There was no way the teacher could set the stage by demonstrating two examples (one of each kind), or by assigning five “seat work” problems and then turning students loose on their homework with a model firmly (for the moment) in place. (de Lange, Burrill, Romberg, & van Reeuwijk, 1993, p. 154).

One of the problems teachers face in approaching instruction from an “explore the environment” perspective is their often incomplete knowledge about the domain. In this regard, Lampert (1988) asked: “How can a teacher who lacks a ‘network of big ideas and the relationship among those ideas and between ideas, facts, and procedures’ develop these things?” (pp. 163–164).

In summarizing a set of studies, I found that the teachers, particularly elementary and middle school teachers, varied in their knowledge of mathematics (Romberg, 1997). Most were elementary-certified teachers with little formal course work beyond the traditional high school algebra and geometry courses taken many years ago. Some had undergraduate degrees in mathematics, but completed several years before. None had ever really done mathematics in the “explore the environment” manner. Little (1993) argued that “[these reforms] represent, on the whole, a substantial departure from teachers’ prior experience, established beliefs, and present practice. Indeed, they hold out an image of conditions of learning for children that their teachers have themselves rarely experienced” (p. 130).

The complex instructional issues involved in creating classrooms that promote understanding include:

- The interconnected roles of tasks, on the one hand, and the manner in which students and their teachers talk about mathematics, on the other
- The ways technological tools can help in the development of classrooms that promote understanding
- The normative beliefs within a classroom about how one does mathematics
- The organizational structures of the classroom
- The role of professional development in helping teachers develop their own classrooms that promote understanding
- The ways the school, as an organization, supports (or impedes) the work of teachers in developing and sustaining such classrooms
- The ways non-school agents (e.g., parents), agencies (e.g., districts), and their actions support (or impede) the development of such classrooms

The vision of reform should ideally focus on this nonroutine pattern of instruction, which allows students to become mathematically literate. However, Susanne Wilson and Robert Floden (2001) found, in a 3-year study in school districts, that although content standards did prove to be a catalyst for some teachers (resulting in a more coherent practice), for most, the rhetoric of reform meant merely mixing the new practices with more ordinary ones.

An example of a research program that helped teachers follow student thinking was Modeling in Mathematics and Science directed by Richard Lehrer and Leona Schauble. In *Investigating Real Data in the Classroom: Expanding Children’s Understanding of Math and*

Science (Lehrer & Schauble, 2002), a collection of stories by teachers involved in the project demonstrates that children can collect and structure data, build their own arguments by appealing to evidence and reason to support claims, and revise models as needed. These teachers were able to document that we have underestimated the capability of students to learn mathematics and science with understanding.

Another approach to examining the problem of nonroutine teaching involves researchers' actually doing the teaching in classrooms. At the elementary grades, one example is the work of Magdalene Lampert and Deborah Ball, who since the early 1980s have been investigating the "pedagogy of investigation" by regularly teaching a class of students. They argue that students learn from "investigating problems, talking with others about potential solutions, building on their own ways of thinking about concepts, and engaging with significant disciplinary ideas" (1999, p. ix). The problem Lampert and Ball have studied is how to teach prospective teachers the pedagogy involved in classrooms that value and practice student investigations. At the secondary level, Dan Chazan (2000) documented 3 years of teaching a lower-track algebra class. He found that there was too little time to get to know students, but if activities helped students see algebraic objects as a part of the world around them, it was possible to get them engaged in learning.

Research in mathematics instruction (in particular, studies such as these) has identified a series of steps that can lead to students' understanding, with each activity justifiable in terms of some potential end points in a learning sequence. The initial instructional activity should make sense to students and be experientially real to them so that they are motivated to engage in personally meaningful mathematical work, raising questions about the problem situation. Although hypothesis generation has rarely been taught, it is a critical aspect of mathematical and scientific reasoning. But as Paul Cobb (1994) noted:

Students' initially informal mathematical activity should constitute a basis from which they can abstract and construct increasingly sophisticated mathematical conceptions. At the same time, the starting point situations should continue to function as paradigm cases that involve rich imagery and thus anchor students' increasingly abstract mathematical activity. (pp. 23–24)

Students' exploring situations embodying a variety of mathematical questions, each of which could be addressed by different methods and described in different ways, will place rigorous demands on teachers' ability to be flexible about the needs of their students. Teachers must not only have an understanding of the mathematics but will need to understand it in a pedagogically reflective way.

Students also need to identify information and procedures they can use to answer their questions. Cobb (1994) argued:

Instructional sequences should involve activities in which students create and elaborate symbolic models of their informal mathematical activity. This modeling activity might involve making drawings, diagrams, or tables, or it could involve developing informal notations or using conventional mathematical notations. (p. 24)

Finally, students need to learn to build coherent cases in support of the quality of their solutions, thereby developing an appreciation of standards of evidence and appropriate forms of

argument. For example, Burrill and I (1998), in a recent study on the learning of statistics, involved students in the work of specifying variables, describing a model to indicate the relationships between variables, and either searching for existing data or collecting new data about the variables. When the students then organized the data to address their questions, they had to identify the differences between categorical data and numerical data. Only after that point did students come to use a variety of descriptive and quantitative techniques in their analyses, including procedures for visualizing the data, calculating data summaries for central tendency and dispersion, and producing tables and scatter plots for pairs of data. Following such descriptions of data, students used inferential techniques to make decisions or predictions based on the data. At this point, the importance of probabilistic reasoning became apparent, but such reasoning remained problematic until sampling and design issues were addressed. Only at the end of this sequence of learning activities on data and data analysis were students finally able to answer the questions originally raised at the start of the sequence.

In summary, today, in contrast to the 1960s, scholars follow a set of assumptions about instruction and schooling practices associated with mathematical literacy. First, all students can and must learn more and somewhat different mathematics than has been expected in the past in order to prepare them to be productive citizens in tomorrow's world. In particular, all students need to have the opportunity to learn important mathematics regardless of socioeconomic class, gender, and ethnicity. Second, we have long underestimated the capability of all students to learn mathematics. Third, some of the important notions we expect students to learn have changed due to changes in technology and new applications. Thus, at every stage in the design of instructional settings we must continually ask, are these ideas in mathematics important for students to understand? Fourth, technological tools increasingly make it possible to create new, different, and engaging instructional environments. With appropriate guidance from teachers, a student's informal models can evolve into models for increasingly abstract mathematical and scientific reasoning. The development of ways of symbolizing problem situations and the transition from informal to formal semiotics are important aspects of these instructional assumptions.

Social context of schooling. Futrell (1986) noted that "education isn't something that any individual does. Education is a *process*, an organic process that involves the complex, dynamic interaction of multiple components" (p. 6). The term *society* was included in Figure 1 to remind all that the teaching and learning of mathematics in school classrooms occurs within a social context that includes administrative policies; community expectations; material, human, and social resources; professional development practices; and so forth. In particular, in the reform contexts of the 1960s and again today, two questions seem appropriate as teachers try to implement changes in their classrooms: What supports and barriers to reform are present in schools and districts? And how can the supports be enhanced and the barriers overcome?

In the 1960s, those involved in the "modern math" movement were aware of the importance of the social context in which teachers worked. In particular, NSF invested heavily in summer and academic-year institutes to provide secondary mathematics teachers with mathematics courses to update their content background. From a modest beginning with a single summer institute for high school teachers in 1954, the program escalated rapidly, until in the late 1960s it supported over 35,000 participants per year and had supported nearly 50% of all secondary science and mathematics teachers (Kreighbaum & Rawson, 1969; Lomask, 1975). The

institutes were extremely popular with Congress because funds went to every congressional district and often to non-elite institutions, which seldom qualified for NSF research and fellowship grants. Nevertheless, by the mid-1970s, questions were being raised about the efficacy of the “top-down” instruction by eminent scientists and their focus on subject-matter expertise to the neglect of pedagogical technique and learning theory. Also, there was no evidence that changes in classroom instruction or student achievement had been effected. The program was discontinued after 1976 (Frechtling, Sharp, Carey, & Vaden-Kiernan, 1995).

Also, during the “modern math” era, answers to the two questions concerning support for and barriers to reform were not of intellectual concern. In the 1969 *Review of Educational Research* issue on mathematics, none of the authors mentioned any research related to either question. Similarly, when Tom Carpenter and I (Romberg & Carpenter, 1986) prepared a review of the research on the teaching and learning of mathematics for the *Third Handbook of Research on Teaching*, these two questions were not addressed, and no research studies in relationship to these questions were identified.

However, by the early 1980s, educational sociologists began studying the organizational context in which most teachers work. They found that when teaching does not change much from day to day (as in most math classes), a predictable flow of material resources—time and curricular material—is the primary support for teaching (Barr & Dreeben, 1983), and the purpose of in-service education is to keep teachers abreast of new materials, techniques, and accountability demands (Fullan, 1993). Teaching for understanding, however, generally involves nonroutine teaching. The usual configuration of resources in schools is not well suited to support an approach to teaching in which teachers use student thinking as a basis for guiding instruction. A predictable flow of material resources, while still important, is not sufficient because this approach to teaching requires new knowledge and collegial ties that are not often easily available (Cohen, McLaughlin, & Talbert, 1993).

Contemporary studies that have directly addressed these issues follow the perspective of Gamoran, Secada, and Marrett (2000) in taking a broad view of resources, identifying teacher development as the primary engine of change, and viewing school organization as a dynamic system. Recently, Fred Newmann and his colleagues (Newmann, King, & Youngs, 2001), based on their prior conception of “authentic human achievement” (Newmann, Secada, & Wehlage, 1995), examined professional development in seven urban schools. They found that “if professional development is to boost school-wide student achievement, it should address five aspects of school capacity: teachers’ knowledge, skills, and dispositions; professional community; program coherence; technical resources; and principal leadership” (p. 290). In the schools they studied, there was considerable variation with respect to each of these aspects, but the “two schools that ranked highest in comprehensiveness . . . both concentrated on improving teaching in literacy and mathematics” (pp. 290–291). In fact, they argued that in “schools that serve low-income students, comprehensive professional development seems necessary to sustain and enhance that capacity” (p. 291).

Adam Gamoran and his colleagues have just completed a 5-year study of the context of changes in teaching in mathematics and science (Gamoran, in press). In six diverse sites, teachers and researchers worked together to enhance teaching for understanding and examined

the school and district contexts of these reform efforts. They concluded that the sustainability of reform required:

- Cultivating social human and material resources by building professional communities, as well as by ensuring resources
- Developing interdependence with other agents, such as the school as a whole, the district, and outside experts, to provide a stable flow of resources
- Using a product regarded as valuable outside the professional community (e.g., evidence of student understanding and achievement)

Using an adaptation of the model developed by Newmann et al. (2001) relating factors that influence school capacity and student achievement, my staff for 3 years provided the mathematics teachers in a local middle school with technical resources, held workshops and meetings to assist the teachers gain new knowledge and skills, and assisted the teachers as they gradually built a professional community and a more coherent program (Romberg, Webb, Burrill, & Ford, 2001):

Overall, there was considerable improvement in the quality of instruction in the classes and in the school's capacity to provide all students with a mathematics program that was comprehensive and meaningful. As teachers' views of student learning of mathematics changed over the course of the project, they experimented with new ways to scaffold student learning during instruction, elicit student responses, and assess student learning. These practices emerged within the context of a school-based collaborative in which administrative leadership, professional community, and technical resources supported teachers' efforts to design classrooms that promote student understanding. (p. 13)

These examples make it clear that studies that approach the two questions raised about the social context in which the teaching and learning of mathematics are situated—that is, questions concerning support for and barriers to reform—are providing researchers insights into the change process.

Research methods. The evolution of research methods in education during the past quarter century has been discussed in several recent reports (e.g., Lagemann, 2000; Lagemann & Shulman, 1999; Shavelson & Towne, 2002), and specifically in mathematics education (e.g., Schoenfeld, 1994, 2001). In the 1960s, most educators used and contributed to quantitative methods used by psychologists (e.g., experimental designs, correlational techniques, psychometrics) as a part of an attempt to build a quantitative science of education. However, as Jerome Bruner (1999) recently pointed out:

Education research as an empirical enterprise should have been quite unproblematic. Its progress might well have been expected to parallel what happened in other forms of “engineering,” where theoretical knowledge is applied to practical problems – like biology applied to medicine or physics to bridge building. But its history, since its beginnings in the latter part of the nineteenth century, has been anything but. It does not seem to have succeeded in the usual way of establishing practices that eventually came to be taken for granted, like vaccination or pasteurization. (p. 399)

The problem, as these authors pointed out, is that schooling is a complex and dynamic social enterprise that does not fit the engineering notions prevalent in other fields. Nevertheless, driven by the spectacular success of experimental methods in the natural sciences, policymakers hope that “by harnessing the logical, conceptual, and computational power of mathematics and statistics, . . . dubious notions about political and social dilemmas might be replaced with carefully reasoned and dispassionately tested scientific inferences” (DeNardo, 1998, p. 125). While the desire for reasoned inferences is understandable, the belief that this can only be done via experimentation is not warranted. In the 1960s, too many quantitative studies lacked any theoretical framework, were too simple in design, involved poor and invalid measurements, and so on. For example, the stereotype of a typical quasi-experiment in education in that era involved the comparison of alternate treatments given to two different groups of students. One treatment was an experimental treatment well described and of interest to the researcher; the other treatment was whatever had been typically done (the conventional treatment). The comparison was made in terms of differences between treatment groups on some posttest. Estimating differences between two treatment effects involved the following set of equations (y = yield; t = treatment; u = unit; and e = error).

$$\begin{array}{ll} y_1 = u_1 + t_1 + e_1 & \text{(measurement after Treatment 1)} \\ \underline{y_2 = u_2 + t_2 + e_2} & \text{(measurement after Treatment 2)} \\ y_1 - y_2 = t_1 - t_2 & \end{array}$$

The difference between the final measurements ($y_1 - y_2$) was considered to be due only to the difference between the effects of the two treatments ($t_1 - t_2$) because it was assumed that the differences in the units and the errors were minimal. One could assume this to be “true” if there was reason to believe that no systematic bias differentiates the experimental units. Such bias is usually controlled by random assignment of subjects to treatment. However, because true control groups are not often feasible in educational settings for a variety of reasons—some of them ethical—any claim of important differences of yield between treatments is logically suspect. When one then adds the often poor documentation of actual differences in treatments as implemented in classrooms and the frequent use of poor or invalid measures, it is no wonder that such research is often seen as problematic, even when well implemented.

Most current research studies do not use experimental comparisons designed to isolate the effect of some treatment. Instead, they involve using empirically developed conceptual frameworks (*causal models* or *local instructional theories*) to explain some phenomenon. The development of such frameworks is in fact what scientific inquiry is about. Stewart, Cartier, and Passmore (2001) pointed out that too often research methods have focused on “a narrowly circumscribed, experimental, perspective on the intellectual activities of scientists” (p. 2). They went on to argue:

Many sciences do not extensively utilize true experiments (the methods of which are idealized in traditional science textbooks) as they conduct empirical inquiries. Ethologists, for example, often collect their data without benefit of control groups. They, like other biologists seeking answers to evolutionary questions, typically employ a nonexperimental, comparative method in their inquiries. If students fail to recognize such methods as ones that are used in scientific inquiry, they surely do not value the significant contributions that a very different form of inquiry makes

to science—inquiries focused on solving conceptual problems (Laudan, 1977). Conceptual problems require that scientists confront the coherence of their “explanations” by examining the internal coherence of explanatory models as well as their relationship to other models, methodological norms, and predominant worldviews of the culture in which science is practiced. It is indeed unfortunate that science is generally portrayed to students as strictly empirical, with little attention given to the deep conceptual issues that have legitimately occupied scientists both past and present. (pp. 4–5)

In fact, reliable and reasoned inferences are more likely from studies that address conceptual issues within an emerging framework, than from comparative experiments.

Today, as Lagemann (2000) and Schoenfeld (2001) pointed out, research methods used by educators (and mathematics educators in particular) tend to be eclectic. These methods involve not just a simple shift from quantitative to qualitative methodology. Instead, as I have described elsewhere (Romberg, 1992a), the methods used to investigate a particular problem should depend on the problem. That the eclectic nature of current research is a concern is reflected in the International Commission on Mathematical Instruction study, *Mathematics Education as a Research Domain: A Search for Identity* (Sierpinska & Kilpatrick, 1998), and in the *Handbook for Research in Mathematics Teaching and Learning* (Grouws, 1992). In particular, given the institutional context of schooling and the cultural diversity of the students whose learning is of interest, teaching experiments (often called design experiments) conducted in classrooms with teachers (again see the examples summarized by Lampert & Ball, 1999; Lehrer & Schauble, 2002)—and involving ongoing interaction of cognitive learning theory and improvement of practice—are critical (Cobb, 2001). Such studies allow scholars to build interpretive frameworks that enable them to analyze students’ learning as it occurs in the social context of the classroom.

In summary, there is considerable agreement among researchers in mathematics education that the work in the 1960s was based on flawed theoretical notions. The mathematical curriculum should not be considered a rutted path, with concepts and procedures to be mastered in some prescribed order. Learning does not simply involve practice, reinforcement, and the shaping of behavior. Teaching should not concern primarily the routine covering of content in classrooms. Instruction cannot ignore the social context in which schools operate classrooms, nor can research scholars investigating instruction rely solely on quantitative experimental methods.

Given the complexity of schooling and the various demands for reform, there is still much disagreement among scholars over the theoretical notions now emerging. In particular, the technical terminology used by some scholars to make their work precise is not used by others, nor do scholars following one conceptual frame often even refer to scholars following another. There is as yet no accepted, shared language available to researchers to characterize work in the field. However, researchers have been aware and concerned about these disagreements. To bring specific issues to the attention of scholars, organizations such as the Wisconsin Center for Education Research, the International Commission on Mathematics Instruction, and the National Council of Teachers of Mathematics have hosted working conferences on specific topics. For example, over 40 scholars from around the world participated in the Wingspread Conference on Addition and Subtraction held in November 1979. Many participants prepared papers that later were published in the book titled *Addition and Subtraction: A Cognitive Perspective* (Carpenter,

Moser, & Romberg, 1982). A number of similar conferences have been held over the past quarter century yielding important scholarly books. Thus, while there are many disagreements in the field, most scholars are aware of the issues even if the disagreements have not been resolved.

Acceptance of Research in the Field

Given that mathematics education is a relatively new area and research on the teaching and learning of mathematics even newer, the following question has gained much importance: By what criteria can research in the field be judged—and by whom? At least three distinct audiences need to be considered: education researchers; mathematics teachers, supervisors, and teacher educators; and mathematicians and other academics.

In the 1960s, there was little mathematics education research around to be accepted by others. Because most programmatic research on mathematics learning was done by psychologists, not mathematics educators (Schoenfeld, 2001), their work in the field was accepted, if at all, by other psychologists. Nor were there any special research sessions on mathematics teaching and learning at AERA meetings until late in the decade. Mathematics teachers and teacher educators also had little use for research. NCTM had only started a research committee in the mid-1960s, and its leaders were reluctant to start research sessions at the annual meetings. The NCTM-sponsored research journal was not initiated until the end of the decade, and then only with considerable reservations. Moreover, mathematicians and other academics paid little attention to mathematics education research: The initial international congresses on mathematics education (1968, 1972) included on their programs many mathematicians, some teacher educators, and a few psychologists, but no mathematics education researchers. Only in 1984 was a mathematics educator finally invited to give a plenary lecture at an International Congress on Mathematics Education meeting.

By the 1990s, there had been considerable change. Within the broader education research community, the work of mathematics educators had become accepted. Mathematics education scholars are now on the editorial boards of most research journals, the special interest group in AERA is strong, and many sessions are given at the annual meetings of the AERA as well as those of NCTM and PME. At international meetings, mathematics educators are now prominent contributors. A mathematics educator has been elected president of AERA, and three are now members of the National Academy of Education.

Mathematics teachers, supervisors, and teacher educators still have much concern about the relevance of education research to ongoing practice in schools, and many feel that much research is carried out and findings disseminated with little input by teachers (National Academy of Sciences, 1999). In his plenary address at the 2001 PME meeting, Jan de Lange argued that “the gap between research and practice is due to the fact that most research around the world has been on the learning of mathematics by individuals and not on schooling practices.” As a consequence, teacher educators find most research papers uninteresting (they deal with marginal details), and teachers cannot synthesize results into useful forms.

Similarly, mathematicians and other academic scholars generally do not value research on the teaching and learning of mathematics in schools, in part because of the low status of all

education research in our society (Lagemann, 2000). Because of its troubled history, education research has not been accepted as an equal and respected partner with the emerging social science disciplines. The efforts during most of the past century focused on creating a behavioral and quantitative science of education, which proved too simplistic. Schooling occurs in complex social institutions that do not lend themselves to the scientific methods used in other fields. For example, the eminent mathematician Lynn Steen (1999), in his review of the report of the ICMI study *Mathematics Education as a Research Domain: A Search for Identity* (Sierpinska & Kilpatrick, 1998), noted that “there is no agreement among leaders in the field about goals of research, important questions, objects of study, methods of investigation, criteria for evaluation, significant results, major theories, or usefulness of results . . . [It is] a field in disarray” (p. 236). Unfortunately, this perception has considerable truth in it. Too many studies still focus on individual learners, not on classroom instruction, and too many studies fail to situate themselves in a chain of inquiry, thus making their importance harder to discern. Only by shifting the unit of investigation from the individual to the classroom or school will the results be useful to teachers; only by identifying how individual studies contribute to important questions will our work make sense to ourselves, let alone to mathematicians and other scholars.

Means of Training New Participants

The health of an academic field in the long run depends on its ability to recruit and train new members to carry on the scholarly traditions in that field. Such preparation usually involves the creation of graduate programs with recruitment policies and procedures, resources, courses, mentors, and practical experiences. In mathematics education, most graduate programs in the past half-century have focused on the preparation of teacher-educators, not education researchers. New graduate students are typically recruited from the ranks of experienced teachers, with courses generally including advanced work in both mathematics and education. Most advanced students gain practical experience through assistantships, working with faculty on mathematics courses, methods courses, and the supervision of preservice teachers. Typically, they are mentored by senior teacher-educators and are expected to conduct small research studies. Schoenfeld (1981), in a review of a collection of dissertations undertaken between 1968 and 1977, found that they lacked theory, suffered from isolationism, and relied too much on statistics. Such studies are learning experiences, but rarely do they also contribute to larger questions in the field. It should be noted that the NSF institutes during the 1960s and 1970s also provided universities the resources to train and to recruit many of today’s mathematics educators (U.S. General Accounting Office, 1984).

Preparation of mathematics education researchers requires courses that include research methods and research seminars and experiences that include participating in research projects as project assistants and being mentored by a researcher. As mentioned earlier, in the 1960s Stanford provided such a program, with its graduate students’ gaining research experience while working on the National Longitudinal Study of Mathematical Abilities.

Today the scene is quite different. Reys, Glasgow, Ragan, & Simms (1999), in their recent survey, found that

- 48 institutions now have doctoral programs in mathematics education;

- the 15 largest programs produced about half of the doctorates from 1980 to 1997; and
- one third of the faculty mentors were graduates of six institutions, all of which had developed a strong research tradition during the past quarter-century.

The programs at these six institutions currently provide the basis for the preparation of new mathematics education researchers. At these institutions, the resources to support graduate students typically come from research grants to senior faculty. Given the dependence of resources on research grants, NSF has periodically directly supported some institutions in granting assistance to promising doctoral students. Of particular importance has been the support of education research training by the Spencer Foundation. Since 1971, the foundation has supported a number of programs to increase the capacity of scholars to study questions relevant to education issues. These programs have included 3-year support to young faculty to develop research agendas, dissertation support for advanced doctoral students, training grants to support graduate students at institutions with a strong research-training base, and so on (Lacy, 1996). Although these programs are not specifically targeted toward mathematics education, many of the participants have focused their work on problems related to the teaching and learning of mathematics.

It also should be noted that NSF recently funded Michigan State University to carry out a “Leadership Development Study” to investigate the characteristics of current leaders in mathematics and science education, the experiences that led them to their positions of leadership, and the kind of preparation the next generation of leaders will need (Gallagher, Floden, Ferrini-Mundy, & Anderson, 2001). Clearly, the preparation of future mathematics education researchers is critical.

Summary

My first claim in this paper was that the number of mathematics educators whose primary job involves research on the teaching and learning of mathematics in schools has increased since the 1960s. I think the supporting evidence is clear: Many scholars and their students now publish results of studies on a regular basis as part of coherent research programs.

My second claim was that as a consequence of the emerging, coherent research programs, a mathematics education research community has emerged. This claim is only partially supported by the evidence presented. Strong evidence exists that today’s scholars no longer hold any of the assumptions underlying the mechanical metaphor for schooling held in the 1960s, but for each of the criteria by which I chose to contrast work in the 1960s with work today, the evidence varies. Today an alternative standards-based vision of school mathematics is generally agreed to, but the details of how this vision can be realized in actual classrooms are not yet apparent. Thus, most classrooms still reflect the “industrial age” view of teaching and learning. Since the 1960s, some additional resources have been made available for conducting research, but those resources are still limited. In contrast to the 1960s, however, scholars now have ample means of sharing information, but still find it difficult to adequately disseminate their findings to teachers, other educators, and policymakers. Today there is some agreement among scholars on theoretical constructs, but as yet little use of a common language to situate current research.

Nevertheless, today knowledge is constructivist rather than formal; the learner is seen as an active participant rather than a product; and teaching is seen as nonroutine rather than routine. Research in mathematics teaching and learning is currently accepted within the education research community, and some inroads have been made with teacher educators and teachers, but there is still little acceptance of this research in other fields. Finally, the preparation of new researchers is now being carried out in a handful of institutions with strong research traditions, available resources, and capable mentors.

Reflections

When preparing this paper, my intent was to marshal evidence that a mathematics education research community had been evolving during the past 30+ years. I am confident that the evidence I have presented confirms the assertion. However, one could ask—so what? Is this not just a paper by an academic justifying his existence? Or, more charitably, one could ask—what has been learned? This is a reasonable question since the purpose of any research community is to make sense of some phenomena, and forming a community of scholars is supposed to expand what is known. Finally, since the emerging picture of progress I have portrayed is very positive, one could ask, is there a “downside” to forming a research community?

What Has Been Learned

It is difficult to summarize what we have learned during the past 30 years of research without getting into the details of specific studies and situating them appropriately. However, I believe at least the following 12 points are clear from the contrast between what was known in the 1960s and what is known today.

First, there are five general findings from mathematics education research that focused on student learning of important mathematics.

1. *We have underestimated the capability of students to learn mathematics with understanding.* Given the opportunity to explore some domain via a set of structured activities, all students can learn important mathematics with understanding. Whether it be measurement of length in kindergarten, or addition and subtraction of whole numbers in Grades 1 and 2, or transformational geometry in Grade 2, or pre-algebra in elementary or middle school, students have little difficulty acquiring the concepts and skills in these domains.
2. *To learn the concepts and skills in a mathematical domain requires that students be engaged in a rich set of structured activities over time.* The discipline of mathematics involves a vast assemblage of ideas in several related content domains. To learn the ideas in a domain, students need the opportunity to investigate problem situations that encourage mathematization. Such situations include those that are subject to measure and quantification, that embody quantifiable change and variation, that involve specifiable uncertainty, that involve our place in space and the spatial features of the world we inhabit and construct, and that involve symbolic algorithms and more abstract structures. In addition, such situations encourage the use of languages for expressing, communicating, reasoning, computing,

abstracting, generalizing, and formalizing. These related systems of signs and symbols extend the limited powers of the human mind in many directions, and they make possible a long-term cultural growth of the subject matter that crosses generations. Finally, such situations embody systematic forms of reasoning and argument to help establish the certainty, generality, consistency, and reliability of mathematical assertions. It is also clear that learning with understanding in a domain is acquired by students gradually over time as a consequence of active engagement in structured activities designed to help students' thinking evolve from informal ideas about a domain to more formal and abstract ways of representing and reasoning in that domain.

3. *Learning with understanding involves more than being able to produce correct answers to routine tasks.* Mathematics should be viewed as a human activity that reflects the work of mathematicians—finding out why given techniques work, inventing new techniques, justifying assertions, and so forth. It should also reflect how users of mathematics investigate a problem situation, decide on variables, decide on how to use mathematics to quantify and relate the variables, carry out calculations, make predictions, and verify the utility of the predictions. Learning with understanding occurs when it becomes the focus of instruction, when students are given time to develop relationships and learn to use their knowledge, and when students reflect about their thinking and express their ideas. Thus, doing mathematics from this perspective cannot be viewed as a mechanical performance or an activity that individuals engage in solely by following predetermined rules.
4. *Modeling and argumentation are important aspects of mathematics instruction that foster learning with understanding.* Modeling, or representing phenomena in the world by means of a system of theoretically specified objects and relations, is critical to the development of understanding in a domain. In classrooms, it is important to consider modeling as a cycle including model construction, model exploration, and model revision. Additionally, as students make conjectures they need to learn to justify their assertions. Thus, argumentation and standards of evidence, with an emphasis on promoting students' skills for generalization in mathematics, are critical.
5. *Student learning should be seen as a product of situative involvement in a classroom culture.* Learning with understanding is a product of interactions over time with teachers and other students in a classroom environment that encourages and values exploration of problem situations, modeling, argumentation, and the like. In fact, the very nature of mathematics is defined communally, making participation by all not only a fundamental civil right, but also important to the continued vitality of mathematics to the nation.

Next, I believe there are four general findings from the research on teaching. As underscored by these findings, the reform approach to teaching represents, on the whole, a substantial departure from teachers' prior experience, established beliefs, and present practice.

6. *Teachers' knowledge of student thinking is critical.* Teachers need to listen and hear what students are saying as they conjecture and build arguments. Teachers also need to examine the work of their students and judge the quality of their justifications and explanations.

7. *Teachers must understand the structure of mathematical domains.* Knowledge of the network of relationships in a domain is critical when making decisions about student understanding and the next steps in instruction.
8. *Rather than just covering the content in a textbook, teachers need to base instruction on the needs of their students.* This finding follows directly from the two preceding findings about teaching. If teachers know the level of their students' thinking and understand how it fits within structure of the domain of interest, then reasonable instruction can be designed.
9. *Professional development cannot be done well in isolation.* Professionalism is the key to quality classroom instruction, but it can only be achieved if teachers join together to collaboratively undertake professional development.

Finally, one of the key aspects of the job of teaching has always been the monitoring of students' progress. Mathematics teachers have traditionally done this by giving quizzes and chapter tests, scoring and counting the number of correct answers on each, and periodically summarizing student performance in a letter grade. Unfortunately, this standard view of assessing students' progress is not consistent with what we know about the learning of mathematics. In fact, the three things we know are about the inadequacy of current procedures.

10. *External tests have an impact on instruction in that teachers take classroom time to prepare students to take the test, but they are not often well related to classroom instruction, nor are the results useful for monitoring growth over time.*
11. *Curriculum-based quizzes and tests are the common mode that mathematics teachers use to gather information and to report grades.* These instruments tend to include items that are very similar to exercises in daily lessons. These tests and quizzes tend to include only items reflecting reproduction, definitions, or computations. They rarely contain items that expect students to relate concepts or to solve nonroutine problems.
12. *Most mathematics teachers are aware that they acquire considerable informal evidence about their students, but rarely use such evidence in judging student progress.* In fact, current data make it clear that most mathematics teachers are poor monitors of student progress.

The “Downside” of Research Communities

The overall strength of a research community lies in the fact that new research can begin where the last left off. Such research can concentrate on subtle or even esoteric aspects of the phenomena, assured that findings will add new information to a conceptual whole. An initial downside for researchers is that they are likely to be castigated for their claims, if the defining features of the emerging community represent a challenge to long-held traditions about schooling. The philosopher of science Thomas Kuhn (1970) has pictured this as a consequence of any scientific revolution. Thus, some public outcry should be expected (e.g., “Isn’t mastery of basic skills enough for most students?” “We should ban calculators.” “Fuzzy math!” “Reading as a math skill—you’ve got to be kidding.” “Multiple-choice items are sufficient.”). Such reactions are not surprising since, in spite of the evidence that the conventional system has not worked well for many students, it has worked for some. Today’s mathematicians and scientists were a

product of the conventional education. So, too, were parents. In fact, many middle-class, high-achieving parents see the current system advantaging their children and are concerned that the proposed changes focusing on all children might harm their children's future (Kohn, 1998).

A second weakness, or downside, of belonging to a research community rests on the fact that adherence to a group and its defining features limits research to the group's shared purpose and vision, thus making questions appear insignificant that may be deemed critical from other perspectives. What if one does not agree with NCTM's reform vision of mathematical literacy? Many thoughtful educators see the reform efforts as based on a "romantic vision" of an unattainable world. Behavioral psychologists view the social constructivist notions of learning as impractical in real school classrooms. Mathematicians concerned about the mathematically talented see the emphasis on "mathematics for all" as a waste of limited resources. And many school administrators, even if convinced of the merit of the vision, see no way of getting there; hiring teachers capable of teaching reform programs and finding resources are beyond their capability. These and other perspectives are clearly counter to the reform perspective, and are not reflected in the research program of the emerging research community. In fact, persons with such perspectives often discount the work of the emerging research community as being "soft," "nonscientific," or "impractical."

Additionally, one could argue that the emerging research community has become too dominant in mathematics education. Its members created the reform vision, they control the major research meetings and publications, they have access to the research resources of the U.S. Department of Education and the National Science Foundation, they train the new participants, and so forth.

Since no research perspective is all-encompassing, a group that develops a perspective and follows it can be very productive and, at the same time, limiting. Throughout the history of science, it has been the emergence of research communities that has led to major contributions to knowledge in specific domains. Such communities have been productive in describing and verifying, over a brief period of time, important findings such as the 12 described above; at the same time, they have been limiting in that other perspectives and questions are not considered.

Conclusion

We have learned a lot about the teaching and learning of mathematics in schools during the past 30+ years, and much of what we have learned are findings of our emerging research community. However, we still have lots to learn about the teaching and learning of mathematics in the "messy" social environment of school classrooms. Laboratory studies on learning mathematics, or design studies with exceptional teachers in affluent suburban schools, are useful but not generalizable. In particular, we have yet to answer the critical question – how can schooling practices in all schools be changed so that they reflect what we have learned?

The fact is that schools are stable social/political organizations that operate via a coherent set of traditions (Schrag, 1981). Changing such organizations involves more than making the changes we believe need to be made in mathematics classrooms. That "more" to be considered involves understanding and dealing with the partisan political and ideological perspectives about

schooling that permeate our society. As a research community, we are only now realizing that the dubious political and social notions held by many about schooling are not easily replaced with carefully reasoned and dispassionately tested inferences. Inferences about schooling procedures based on research findings have yet to replace ideological inferences in political spheres because, it is argued, either the research is based on grossly unrealistic reductions of complex phenomena or the research involves conflicts of values that cannot be resolved by evidence. In fact, it is naïve to believe that we can curtail partisan prejudices about schooling by drawing on research findings. Nevertheless, we must continue the effort.

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